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Software in the Wolfram Language for Real Algebraic Curves

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Mathematica functions are given to create, transform and represent real curves given by systems of polynomials with, possibly, numeric coefficients using mostly numerical algorithms. These are divided into three parts, plane curves given by a single bivariate polynomial, curves in $\mathbb{R}^3$ defined by 2 polynomials in 3 variables and curves in $\mathbb{R}^n, n \geq 3$ defined by a system of $n-1$ or more equations. Although the curves are given in affine form ultimately they are considered as projective curves. The transformations are projective linear transformations, sometimes called linear fractional transformations or fractional linear transformations.

The first case of plane curves is given in my book [1,2] and/or freely available Wolfram notebooks [1,3]. In addition to well known curves, constructions are given for two classes of real curves, one given by Gauss in his 1799 proof of the Fundamental Theorem of Algebra the other motivated by Newton's work on cubic curves. Curves are analyzed by finding critical and infinite points and tracing paths between them. Because the projective plane is compact this is a finite process. Representations include plotting on the Möbius band. All code is available at [3].

The other two cases in this poster are from a second volume currently in the writing phase. A summary of what is available now is given in [3] along with the code. The case of 2 polynomials in $\mathbb{R}^3$ is similar to the plane case thanks to the availability of the cross product. The last case is handled by projecting to the plane, analyzing as in the first case, and lifting, all using lots of numerical linear algebra.

The one general concept running through all cases is the *Fundamental Theorem* which states that each algebraic real curve can be represented by a graph with vertices certain projective points on the curve and edges non-singular paths between vertices. These graphs have the property that each vertex is even.

Keywords
Algebraic Curves, Numerical Algebraic Geometry, Wolfram Language

References

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[2] Barry H Dayton, A Wolram Language Approach to Numerical Algebraic Plane Curves *The Mathematica Journal* **20**(August 29, 2018), Free PDF from `mathematica-journal.com`.
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